The illustrative examples of the elementary short questions recommended by the Ad Hoc Committee on the Extended Part of Senior Secondary Mathematics (Enhanced Measures for Catering Learner Diversity)

M2 suggested elementary short questions

1. Let $y = e^{3x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ from first principles.

2. (a) Using mathematical induction, prove that

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \text{ for all positive integers } n.$$

(b) Using (a), evaluate $3^3 + 6^3 + 9^3 + \cdots + 300^3$.

3. Let n be a positive integer. The coefficient of x^2 in the expansion of $(1+3x)^n(1-2x)^2$ is 160. Find

(a) n,

(b) the coefficient of x^3 in the expansion.

4. (a) Using integration by substitution, find $\int \frac{x}{e^{x^2}}\,\mathrm{d}x$.

(b) At any point (x, y) on the curve C, the slope of the tangent to C is $\frac{3x}{e^{x^2}}$. If the point $(0, 1)$ lies on C, find the equation of C.

5. (a) Prove that $\frac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \tan\beta$.

(b) Suppose that $\sin(x+y) - 2\cos(x+y) = 2\cos(x-y) + \sin(x-y)$ for some real numbers x and y. Using (a), or otherwise, find the value of $\tan 2y$.

6. Define $\mathrm{f}(x) = \frac{3x^2-x+2}{x+1}$ for all $x \neq -1$. Denote the graph of $y = \mathrm{f}(x)$ by *C*. Find

(a) the asymptote(s) of *C*,

(b) the equation of the tangent to *C* at the point with the *x*-coordinate -2.

7. Consider the curve *C*: $y = \cos x \sin 2x$, where $0 \leq x \leq \frac{\pi}{2}$.

Find the extreme point(s) of *C*.

8. (a) Express $(\sec x - \cos x)^2$ in the form of $\sec^2 x + p\cos 2x + q$, where p and q are constants.

(b) Using the result of (a), or otherwise, evaluate $\int_0^{\frac{\pi}{4}} (\sec x - \cos x)^2 \mathrm{d}x$.

9. (a) Find $\int x\ln 2x \,\mathrm{d}x$.

(b) Consider the curve *C*: $y = \sqrt{x\ln 2x}$, where $x \geq \frac{1}{2}$. Let *R* be the region bounded by *C*, the *x*-axis and the straight line *x* = 5. Find the volume of the solid generated by revolving *R* about the *x*-axis.

10. *M* is a point lying on *XY* such that *XM* : *MY* = 1 : 2. Let $\overrightarrow{OX} = \mathbf{x}$ and $\overrightarrow{OY} = \mathbf{y}$, where *O* is the origin.

(a) Express $\overrightarrow{OM}$ in terms of **x** and **y**.

(b) It is given that $|\mathbf{x}| = 2$, $|\mathbf{y}| = 3$ and $\mathbf{x}\cdot\mathbf{y} = 2$. Using the result of (a), find $|\overrightarrow{OM}|$.

11. Let $\overrightarrow{OA} = -\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{j} + 5\mathbf{k}$.

(a) Evaluate $\overrightarrow{OA} \times \overrightarrow{OB}$.

(b) Using the result of (a), or otherwise, find the angle between *OC* and the plane *OAB*.

12. Consider the following system of linear equations in real variables *x*, *y*, *z*

$$(E): \begin{cases} 3x + y + z = 1 \\ -x + az = b \\ 2x + y + 3z = 2 \end{cases}$$, where *a* and *b* are real constants.

(a) Assume that (*E*) has a unique solution.
   (i) Find the range of values of *a*.
   (ii) Express *y* in terms of *a* and *b*.

(b) Assume that (*E*) has infinitely many solutions. Solve (*E*).

13. Let $A = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$.

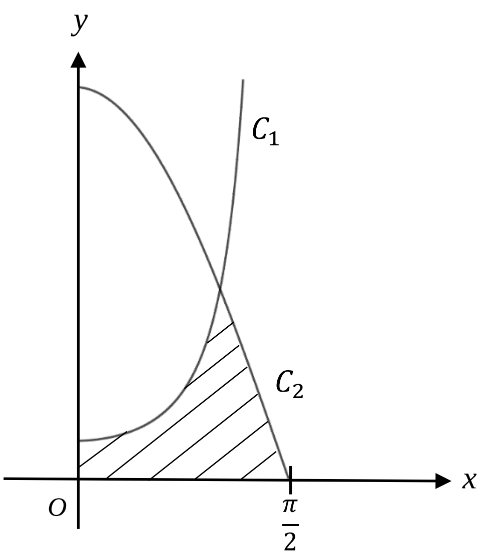
(a) Find B^{-1}.

(b) Find a matrix *C* such that *AB* = *BC*.

(c) Using the result of (b), find A^{100}.

14. (a) Solve the equation $\sec^2 x - 8\cos x = 0$, where $0 \leq x < \frac{\pi}{2}$.

(b) The following figure shows the shaded region bounded by the curve C_1: $y = \sec^2 x$ and the curve C_2: $y = 8\cos x$, where *O* is the origin. Find the area of the shaded region.



15. The coordinates of the points *A* and *B* are (−1, 0) and (5, 0) respectively. The point *C* moves upwards along the *y*-axis from the origin *O* such that the perimeter of ΔABC increases at a constant rate of $\frac{1}{7}$ units per second.

(a) Find the rate of change of *OC* when *OC* = $2\sqrt{6}$ units.

(b) Let $\angle ABC = \theta$ radians.

(i) Using the result of (a), find the rate of change of θ when *OC* = $2\sqrt{6}$ units.

(ii) Find the rate of change of θ when $AC \perp CB$.